

# Parity of $\Theta^+(1540)$ from QCD sum rules

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## Abstract

The QCD sum rule for the pentaquark  $\Theta^+$ , first analyzed by Sugiyama, Doi and Oka, is reanalyzed with a phenomenological side that explicitly includes the contribution from the two-particle reducible kaon-nucleon intermediate state. The magnitude for the overlap of the  $\Theta^+$  interpolating current with the kaon-nucleon state is obtained by using soft-kaon theorem and a separate sum rule for the ground state nucleon with the pentaquark nucleon interpolating current. It is found that the K-N intermediate state constitutes only 10% of the sum rule so that the original claim that the parity of  $\Theta^+$  is negative remains valid.

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## I. INTRODUCTION

The discovery of the  $\Theta^+$  baryon by LEPS Collaboration at SPring-8 [1] and subsequent confirmation in the other experiments [2] have spurred a lot of works in the field of exotic hadrons. So far, not much is known about the properties of the  $\Theta^+$  except its mass, which is about 1540 MeV, and its small decay width, which is smaller than the experimental resolutions of around 10 MeV. Thus, to determine the quantum numbers as well as other properties of  $\Theta^+$ , various production mechanisms have been proposed [3]. The  $\Theta^+$  baryon, being a strangeness +1 state, is exotic since its minimal quark content should be  $uudd\bar{s}$ . Other states that have positive strangeness but different charges are not observed<sup>1</sup>, which suggests that the  $\Theta^+$  is an isosinglet. The existence of such an exotic state with narrow width and spin-parity  $1/2^+$  was first predicted by Diakonov *et al.* [5] in the chiral soliton model, where the  $\Theta^+$  is a member of the baryon anti-decuplet. The positive parity is also supported by the constituent quark model with flavor-spin hyperfine interaction [6], the diquark-diquark-antiquark picture of Jaffe and Wilczek (JW) [7], the triquark-diquark picture [8], the quark potential model calculations [9], and the constituent quark model where chiral dynamics are included [10]. On the other hand, it is expected in a naive constituent quark model that the ground state of the pentaquark have a negative parity because all the quarks would be in the  $s$ -state. The negative parity is supported by the calculations based on QCD, such as the lattice calculation [11] and QCD sum rules [12]. Hence, determining the parity of the pentaquark states will not only be important in establishing the basic quantum numbers of the pentaquark states, but also in understanding the QCD dynamics especially when multiquarks are involved.

Currently the results from both lattice QCD [11] and QCD sum rule [12] analysis, which show the existence of a negative parity pentaquark state in the isospin zero and spin  $1/2$  channel, face a challenge to be settled. In particular, subsequent analysis in the lattice QCD found no stable pentaquark state in the advertised channel [13]. Similarly, in a different QCD sum rule analysis, the parity was found to be positive [14]. A major uncertainty in both approaches is associated with isolating the pentaquark contribution in the correlation functions between the pentaquark interpolating currents. Because the interpolating current can also couple to the two-hadron reducible (2HR) kaon-nucleon (K-N) intermediate state, it is difficult to extract signals for the pentaquark state from the theoretical calculation of the two point correlation function. This is particularly so because the K-N threshold lies below the expected  $\Theta^+$  state and neither the Borel transformation in the QCD sum rule nor the large imaginary time behavior in the lattice calculation can isolate the  $\Theta^+$  state. Hence, it is essential in both approaches to estimate the contribution coming from the K-N intermediate state in the correlation function.

This point has been noted for the QCD sum rule approach by Kondo *et al.* [14], who claimed that after subtracting out the 2HR part of the operator product expansion (OPE), one finds that the parity becomes positive. However, as we will show, subtracting the 2HR contribution in the OPE level is an ill-defined approach. Instead, its contribution can be estimated in the phenomenological side. The magnitude for the overlap of the  $\Theta^+$  interpolating current with the kaon-nucleon state is obtained by first applying a chiral rotation to the  $\Theta^+$  interpolating current and estimate the kaon overlap in the soft-kaon limit. We then analyze the QCD sum rule for the nucleon with the resulting pentaquark nucleon

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<sup>1</sup> For various classifications of the pentaquarks and their decay modes, see Ref. [4].

interpolating current to estimate the nucleon overlap. It is found that the K-N intermediate state constitutes only 10% of the sum rule so that the original claim that the parity of  $\Theta^+$  is negative remains valid.

This paper is organized as follows. In section II, we present our method of treating the K-N intermediate state appearing in the correlation function for the  $\Theta^+$  sum rule. We then calculate the overlap of the  $\Theta^+$  interpolating field with the K-N intermediate state in Section III. In section IV, we reanalyze the  $\Theta^+$  sum rule after subtracting out the K-N intermediate state.

## II. CORRELATION FUNCTION

Let us begin with the correlation function between the interpolating field for  $\Theta^+$ ,

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_\Theta(x), \bar{J}_\Theta(0)] | 0 \rangle \quad (1)$$

where

$$\begin{aligned} J_\Theta &= \epsilon^{abc} \epsilon^{def} \epsilon^{cfg} \{u_a^T C d_b\} \{u_d^T C \gamma_5 d_e\} C \bar{s}_g^T, \\ \bar{J}_\Theta &= -\epsilon^{abc} \epsilon^{def} \epsilon^{cfg} \{\bar{d}_e \gamma_5 C \bar{u}_d^T\} \{\bar{d}_b C \bar{u}_a^T\} s_g^T C. \end{aligned} \quad (2)$$

Here the roman indices denote the color,  $C$  the charge conjugation and the superscript  $T$  transpose. The OPE of this correlation function has been calculated by Sugiyama, Doi and Oka (SDO)[12] and its extension to the anti-charmed pentaquark has been made in Ref.[15]. From comparing the OPE to the phenomenological side saturated by the ground state  $\Theta^+$  and a continuum, SDO were able to identify the parity of the  $\Theta^+$  to be negative.

However, as has been noted by Kondo, Morimatsu and Nishikawa (KMN)[14], the correlation function can have two-hadron reducible (2HR) contributions in addition to the two-hadron irreducible (2HI) part. This means that since  $J_\Theta$  is the 5 quark current with a strangeness +1, isospin zero, it can also easily couple directly to a kaon-nucleon intermediate state or any of their excited states, namely,

$$\Pi(q) = \Pi^{2HI} + \Pi^{2HR} \quad (3)$$

where

$$\begin{aligned} \Pi^{2HI} &= -\frac{|\lambda_\Theta|^2}{\not{q} - m_\Theta} \dots \\ \Pi^{2HR} &= -i \int \frac{d^4p}{(2\pi)^4} \Pi_N(p) \Pi_K(p - q). \end{aligned} \quad (4)$$

Therefore, to extract information about the pentaquarks from the OPE calculation, one has to subtract the contributions from the 2HR contributions,

$$\begin{aligned} \Pi^{2HI}(q) &= \Pi^{OPE}(q) - \Pi^{2HR}(q) \\ &= \Pi^{OPE}(q) + i \int \frac{d^4p}{(2\pi)^4} \Pi_N(p) \Pi_K(p - q). \end{aligned} \quad (5)$$

In the left hand side (LHS), we are interested in the ground state but the OPE part can be calculated for large  $-q^2$ . The usual way of matching the two sides with different regions of the momentum is achieved by the Borel transformation. A question in this particular case is how to subtract the 2HR contribution effectively.

| contribution to $\Pi^{2HR,OPE}(q)$ | $\Pi_N^{OPE}(p)$ | $\Pi_K^{OPE}(p-q)$ | comments      |
|------------------------------------|------------------|--------------------|---------------|
| large $-q^2$                       | large $-p^2$     | large $-(p-q)^2$   | in Eq.(6)     |
| large $-q^2$                       | small $-p^2$     | large $-(p-q)^2$   | not in Eq.(6) |
| large $-q^2$                       | large $-p^2$     | small $-(p-q)^2$   | not in Eq.(6) |

TABLE I: Typical momentum regions which contribute to the OPE of  $\Pi^{2HR}(q)$ . The first line represents the region which has been taken into account through Eq.(6).

### A. Method by KMN

KMN suggest to calculate the large  $-q^2$  limit of  $\Pi^{2HR}(q)$  using the OPE of the  $\Pi_N^{OPE}(p), \Pi_K^{OPE}(p-q)$ , namely

$$\Pi^{2HR,OPE}(q) = -i \int \frac{d^4p}{(2\pi)^4} \Pi_N^{OPE}(p) \Pi_K^{OPE}(p-q) . \quad (6)$$

Under this prescription, KMN found that the contribution from the 2HR is large enough to change the previous result on the  $\Theta^+$  parity. However, a little inspection shows that such factorization is ill-defined. The reason is the following. The OPE of the LHS of Eq.(6) means that it is obtained from the short-distance expansion of the correlator; namely in the large  $-q^2$  limit. Also being the OPE parts,  $\Pi_N^{OPE}(p)$  and  $\Pi_K^{OPE}(p-q)$  are obtained in the large  $-p^2$  and  $-(p-q)^2$  limit respectively. However, there are other important regions of  $p^2$  which contribute to the OPE of the LHS. An example of such regions are given in table I. Another serious problem with Eq.(6) is the implicit assumption of

$$\langle 0 | J_K J_N | KN \rangle = \langle 0 | J_K | K \rangle \times \langle 0 | J_N | N \rangle, \quad (7)$$

which can be shown to be not true in general.

### B. Our method

Here, we suggest to subtract out the 2HR contribution by explicitly estimating the contribution coming from the non-interacting K-N intermediate state,

$$\Pi^{2HR}(q) = i |\lambda_{KN}|^2 \int \frac{d^4p}{(2\pi)^4} \frac{\gamma_5 (\not{p} + m_N) \gamma_5}{p^2 - m_N^2} \frac{1}{(p-q)^2 - m_K^2} \quad (8)$$

where

$$\langle 0 | J_\Theta | KN(p) \rangle = \lambda_{KN} i \gamma_5 u_N(p) . \quad (9)$$

There are additional contributions coming from excited kaon or nucleon states. However, these contributions are exponentially suppressed after the Borel transformation. Hence, to estimate the lowest 2HR contribution, we need to know the overlap strength in Eq.(9). This strength will be estimated in the following section by combining the soft-kaon limit and a sum rule for the nucleon with pentaquark interpolating field.

### III. ESTIMATING THE OVERLAP STRENGTH

To calculate the overlap strength  $\lambda_{KN}$ , we first use the soft-kaon theorem,

$$\begin{aligned} \langle 0|J_\Theta|KN\rangle &\xrightarrow{\text{soft-kaon}} -\frac{1}{f_K}\langle 0|[Q_5^K, J_\Theta]|N\rangle = -\frac{1}{f_K}\langle 0|J_{N,5}|N\rangle \\ &= -\frac{\lambda_N}{f_K}i\gamma_5 u_N(p) \end{aligned} \quad (10)$$

where  $Q_5^K = \int d^3y d^\dagger(y) i\gamma_5 s(y)$  and

$$\begin{aligned} J_{N,5} = & \epsilon^{abc}\epsilon^{def}\epsilon^{cfg}\left[\{u_a^T C \gamma_5 s_b\}\{u_d^T C \gamma_5 d_e\}C\bar{s}_g^T + \{u_a^T C d_b\}\{u_d^T C s_e\}C\bar{s}_g^T \right. \\ & \left. + \{u_a^T C d_b\}\{u_d^T C \gamma_5 d_e\}C\gamma_5 \bar{d}_g^T\right]. \end{aligned} \quad (11)$$

Using Eq.(10) in Eq.(9) we have,

$$\lambda_{KN} = -\frac{1}{f_K}\lambda_N. \quad (12)$$

Hence, to estimate  $\lambda_{KN}$ , we need to calculate  $\lambda_N$  which represents the five-quark component of the nucleon.

To do that, we first construct the sum rule for the nucleon using the following “old-fashioned” correlation function,

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0|\theta(x^0)J_{N,5}(x)\bar{J}_{N,5}(x)|0\rangle, \quad (13)$$

where  $J_{N,5}$  is given in Eq.(11). This type of “old-fashioned” correlation function has been successfully used in projecting out positive and negative parity nucleon [16]. We then divide the imaginary part into two parts for  $q_0 > 0$ ,

$$\frac{1}{\pi}\text{Im}\Pi(q_0) = A(q_0)\gamma^0 + B(q_0). \quad (14)$$

Then the spectral density for the positive and negative parity physical states will be as follows,

$$\rho^\pm(q_0) = A(q_0) \mp B(q_0). \quad (15)$$

Note that the signs are reversed compared to that of SDO because the nucleon current  $J_{N,5}$  as given in Eq.(11) has an additional factor of  $\gamma_5$  compared to the usual nucleon current.

For the nucleon correlation function given in Eq.(13), the respective OPE are given by

$$A_{OPE}(q_0) = \frac{3q_0^{11}}{5!5!2^{10}7\pi^8} + \frac{4q_0^7 m_s \langle \bar{s}s \rangle}{3!5!2^8\pi^6} + \frac{3q_0^7}{3!5!2^{10}\pi^6} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle - \frac{4q_0^5}{3!4!2^9\pi^6} m_s \langle \bar{s}g\sigma \cdot Gs \rangle \quad (16)$$

$$B_{OPE}(q_0) = \frac{2q_0^{10}m_s}{5!5!2^{10}\pi^8} - \frac{q_0^8 [2\langle \bar{s}s \rangle - \langle \bar{d}d \rangle]}{4!5!2^7\pi^6} + \frac{q_0^6}{3!4!2^9\pi^6} [2\langle \bar{s}g\sigma \cdot Gs \rangle - \langle \bar{d}g\sigma \cdot Gd \rangle]. \quad (17)$$

Here, we have kept the extra numeric factors in the numerator so that our OPE can be directly compared with that of the SDO sum rule. From the comparison, we see that the

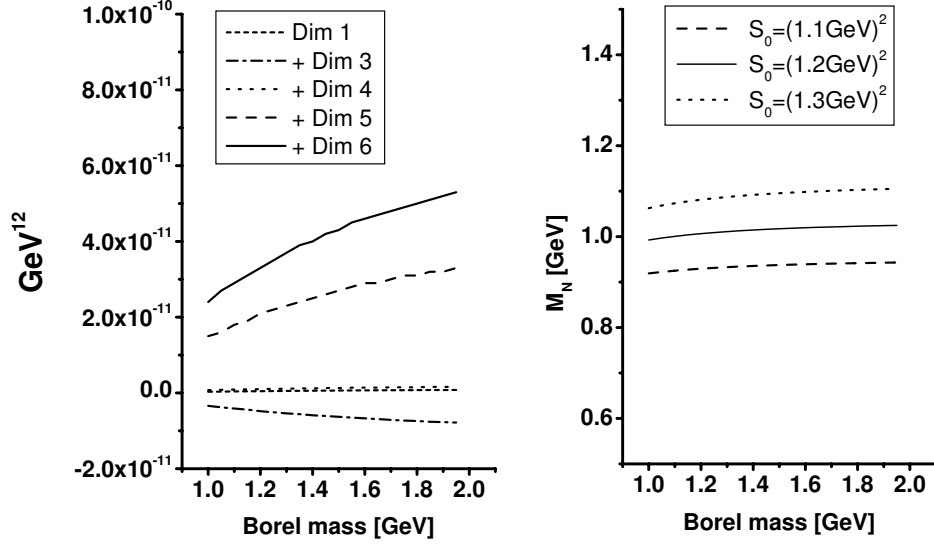


FIG. 1: The figure in the left panel shows the Borel curves for  $|\lambda_{N\pm}|^2 e^{-m_{N\pm}^2/M^2}$  from Eq. (20) as we add up the OPE order by order. The figure in the right panel shows the mass of the nucleon from the five-quark current with specified continuum threshold.

dimension-even operators have been amplified by the factor 3 or 4 while for the dimension-odd operators (dimension 3 and 5) there are partial cancellations among them.

The spectral density is assumed to have the following form,

$$\rho_{phen}^{\pm}(q_0) = |\lambda_{N\pm}|^2 \delta(q_0 - m_{N\pm}) + \theta(q_0 - \sqrt{s_0}) \rho_{cont}^{\pm}(q_0) , \quad (18)$$

where the usual duality assumption has been used to represent the higher resonance contribution above the continuum threshold  $\sqrt{s_0}$ . We substitute this into the following Borel transformed dispersion relation,

$$\int_0^{\infty} dq_0 e^{-q_0^2/M^2} [\rho_{phen}^{\pm}(q_0) - \rho_{OPE}^{\pm}(q_0)] = 0 , \quad (19)$$

and obtain a sum rule for  $|\lambda_{N\pm}|^2$

$$|\lambda_{N\pm}|^2 e^{-m_{N\pm}^2/M^2} = \int_0^{\sqrt{s_0}} dq_0 e^{-q_0^2/M^2} \rho_{OPE}^{\pm}(q_0) . \quad (20)$$

The sum rule for the nucleon mass is obtained by taking the derivative with respect to  $1/M^2$ . Using the same QCD parameters as in Ref.[12], we plot the RHS of Eq.(20) and the Borel curve for the nucleon mass in Fig.1. As can be seen from the figure in the left panel, the OPE with dimension 5, which contains the quark-gluon mixed condensate, still contributes to the sum rule appreciably. This feature is similar to the SDO sum rule where the quark-gluon mixed condensate is the main origin for yielding the negative-parity. However, in our case, we have additionally important contribution from the dimension 6 operator. Because of this,

the pentaquark nucleon current does not exclusively couple to a specific parity state but it couples to both parities. In fact, from Fig.1, we obtain a consistent (positive) sum rule for  $|\lambda_{N+}|^2 \sim 1 \times 10^{-10} \text{ GeV}^{12}$  and a reasonable mass for the nucleon. Similarly we also obtain consistent results for the negative-parity nucleon  $S_{11}(1535)$  from the sum rule for  $|\lambda_-|^2$ .

#### IV. REANALYSIS OF SDO SUM RULE

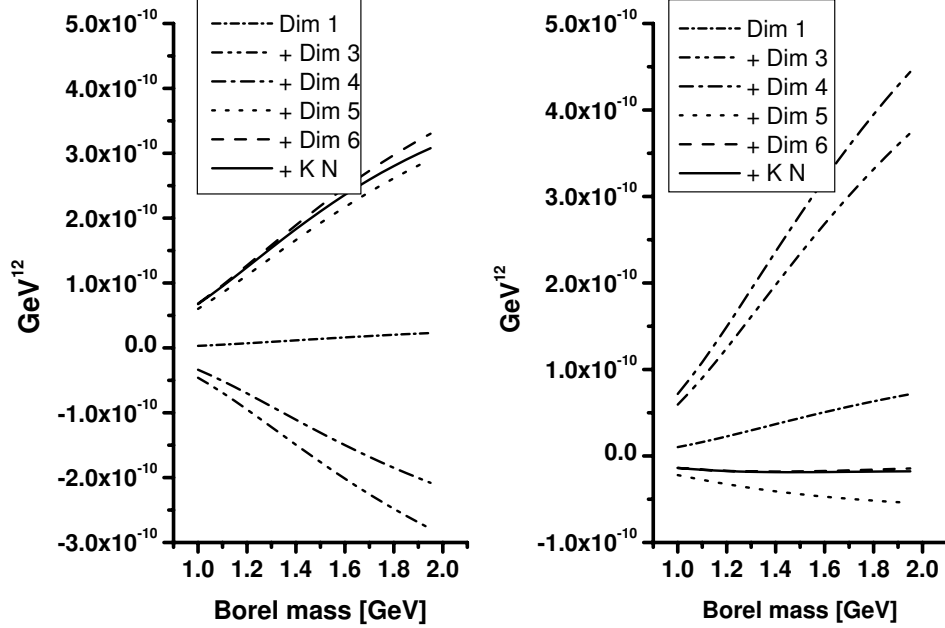


FIG. 2: Borel curves for the LHS of Eq. (22) for the negative-parity (left panel) and positive-parity (right panel) pentaquark as we add up the OPE order by order. The solid lines are obtained when the K-N 2HR contribution is subtracted from the OPE.

We now reanalyze the SDO sum rule including the K-N 2HR contribution given by Eq.(8). The imaginary parts of Eq.(8) for the positive and negative parity channels are

$$\rho_{KN}^{\pm}(q_0) = \frac{|\lambda_{KN}|^2}{32\pi^2} \sqrt{(q_0 - m_K)^2 - m_N^2} \sqrt{(q_0 + m_K)^2 - m_N^2} \times \frac{(q_0 \mp m_N)^2 - m_K^2}{q_0^3}. \quad (21)$$

Here we use  $|\lambda_{KN}|^2 = |\frac{1}{f_K} \lambda_{N+}|^2$  as determined from the previous section. Then, the SDO sum rule with the explicit contribution from the K-N 2HR contribution subtracted out reads

$$|\lambda_{\Theta\pm}|^2 e^{-\frac{M_{\pm}^2}{M^2}} = \int_0^{\sqrt{s_0}} dq_0 e^{-\frac{q_0^2}{M^2}} \rho_{OPE}^{\pm}(q_0) - \int_{m_K+m_N}^{\infty} dq_0 e^{-\frac{q_0^2}{M^2}} \rho_{KN}^{\pm}(q_0) \quad (22)$$

where  $\rho_{OPE}^{\pm}$  is given in Ref.[12]. Note, the LHS is positive definite. The reliable sum rule should have the RHS with the same sign. The RHS of Eq. (22), which includes the OPE as

well as the K-N 2HR contribution, is plotted in Fig.2. We see that for the negative-parity case the RHS is positive agreeing with the sign in the LHS. But for the positive-parity case, the sign of the RHS does not satisfy the constraint on sign required by the LHS. As can be seen also in the figure, the contribution from the 2HR state constitutes less than 10% of the total OPE. Hence, the conclusion first given by SDO that the OPE is consistent with the existence of a negative parity pentaquark state remains valid.

In summary, we have reanalyzed the QCD sum rule for  $\Theta^+$  with the K-N 2HR contribution being subtracted out. The strength for the K-N 2HR with the  $\Theta^+$  interpolating field has been estimated by the soft-kaon theorem and the resulting nucleon sum rule with five-quark current. The K-N continuum contribution was found to be less than 10 % and the QCD sum rule for  $\Theta^+$  still supports a negative-parity state.

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